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QCD effects and b -tagging at LEP I¹

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Abstract

We analyze the impact of using b -tagged samples in studying non-Abelian effects due to QCD in $e^+e^- \rightarrow 4\text{jet}$ events at $\sqrt{s} = M_{Z^0}$, using angular variable analyses and comparisons with $e^+e^- \rightarrow 3\text{jet}\gamma$ events. We find that QCD effects are largely enhanced in b -quark samples with respect to ‘unflavoured’ ones, where energy-ordering is used to distinguish between gluon and quark jets. We show that the b -quark mass influences the angular distributions significantly and should not be neglected.

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1 Introduction

In recent years the experiments ALEPH, DELPHI, L3 and OPAL at LEP I have performed a number of measurements of the process $e^+e^- \rightarrow Z^0 \rightarrow \text{hadrons}$ in order to point out effects due to QCD [1]. Important results have been achieved. The strong coupling constant α_s has been determined from jet rates and from shape variables [2] and both the flavour independence [3] and the running with \sqrt{s} [4] have been verified. Three- [5] and four-jet [6] distributions have been studied and their behaviour agrees with QCD predictions calculated to second order in α_s . The colour factors, which determine the gauge group responsible for strong interactions, have been measured [7]. Abelian models alternative to QCD have been ruled out and the coupling of the QCD triple gluon vertex has been verified to be in agreement with QCD predictions [8].

Concerning the latter tests, several variables sensitive to differences between QCD and Abelian models have been proposed [9, 10, 11, 12, 13]. One of the main differences is the predicted relative contribution of $e^+e^- \rightarrow q\bar{q}q'\bar{q}'$ events in $e^+e^- \rightarrow 4\text{jet}$ samples: about 5% in QCD but 30% in Abelian models [14]. The tests are preferentially based on angular correlations between jets, which are usually ordered in energy and where the two most energetic jets are “identified” with the primary quarks (i.e., from the Z^0 decay). Among these angular variables, some of the most widely used in experimental analyses are the modified Nachtmann-Reiter angle θ_{NR}^* [10, 11], the Bengtsson-Zerwas angle χ_{BZ} [12], the Körner-Schierholz-Willrodt angle Φ_{KSW} [13] and the angle between the two least energetic jets θ_{34} . The distributions in these variables are quite different for $e^+e^- \rightarrow q\bar{q}gg$ and $e^+e^- \rightarrow q\bar{q}q'\bar{q}'$ events.

Another way of searching for evidence of effects due to QCD, and only partially exploited so far, is to use photon samples. In particular, in order to isolate the triple gluon vertex contribution, one can compare 4jet- and 3jet γ -samples. This approach is based on the similarity of photon and gluon bremsstrahlung off quarks [15] and has already been adopted in ref. [16]. However, there is also an obvious difference between photons and gluons, which is that only photons can also be radiated off the initial state electrons and positrons. Therefore some care is needed to eliminate the distortion due to this Initial State Radiation (ISR) from the 3jet γ -sample before a direct comparison with the 4jet sample can be made.

It is the purpose of this paper to study to what extent the techniques of flavour identification, that are rapidly being developed by the various experimental collaborations at LEP I [17], turn out to be useful in recognizing effects of QCD from the analysis of b -tagged 4jet- and 3jet γ -samples, following both the approaches described above. The most widely used methods to recognize b -quark jets are probably the following:

- reconstruction of semileptonic b -decays by observing a high p_T lepton;
- lifetime tagging by detecting a secondary vertex;
- reconstruction using kinematical “event shape” or “jet shape” variables.

Their main features are summarized in refs. [18, 19]. Recently, in addition to these and other conventional methods [20] also the identification of gluon and quark jets by means of neural networks has been proposed [21].

In our opinion, there is an important motivation for this study. As many authors have pointed out, the angular variables described here are most useful for emphasizing the non-Abelian features of QCD if one distinguishes between quark and gluon jets and assigns the

four-momenta of the final states to the corresponding particles. If that is not possible, the best one can do is to order the jets in energy². However, with b -tagging, it is at least possible to distinguish some quark jets, namely those originating from b -quarks, from gluon jets. This opens the prospect of observing the non-Abelian structure of QCD much more clearly by selecting events containing b -jets. Since this means that all final states contain at least two b -quarks, the effect of the b -quark mass becomes much more important than it was before, and it must be properly taken into account in the analysis. The required matrix elements have only recently become available [22, 23, 24]³.

The plan of the paper is as follows. In section 2 we give details of the calculation, of the algorithms used in the phenomenological analysis and the numerical values adopted for the various parameters. Section 3 is devoted to a discussion of the differential distributions in four angular variables, and section 4 to the possibility of using 3jet γ samples. In section 5 we study the sensitivity of our results to the b -quark mass, and we draw our conclusions in section 6.

2 Calculation

The Feynman diagrams describing at tree-level the reactions

$$e^+ + e^- \rightarrow q_1 + \bar{q}_2 + q'_3 + \bar{q}'_4, \quad (1)$$

$$e^+ + e^- \rightarrow q_1 + \bar{q}_2 + g_3 + g_4, \quad (2)$$

$$e^+ + e^- \rightarrow q_1 + \bar{q}_2 + g_3 + \gamma_4, \quad (3)$$

where $q^{(\prime)} = u, d, s, c$ and b , are shown in figs. 1-2. In the present analysis we have computed the corresponding matrix elements with the same FORTRAN code already used in refs. [22, 23, 24], which takes all masses and both the γ and Z^0 intermediate contributions into account exactly. For all the details of the computation, as well as for the explicit helicity amplitude formulae, we refer to [23].

The matrix element squared $|\mathcal{M}|^2$ for the four quark process (1) can be written as

$$|\mathcal{M}|^2 = C_+^{q\bar{q}q'\bar{q}'} |\mathcal{M}_+|^2 + C_-^{q\bar{q}q'\bar{q}'} |\mathcal{M}_-|^2, \quad (4)$$

with

$$C_{\pm}^{q\bar{q}q'\bar{q}'} = \frac{1}{2} N_C C_F (T_F \mp (C_F - \frac{1}{2} C_A)), \quad (5)$$

$$\mathcal{M}_{\pm} = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 + \mathcal{M}_4 \pm \delta^{qq'} (\mathcal{M}_5 + \mathcal{M}_6 + \mathcal{M}_7 + \mathcal{M}_8), \quad (6)$$

where \mathcal{M}_i is the amplitude corresponding to the i -th diagram in fig. 1, $N_C = 3$ the number of colours, $C_F = (N_C^2 - 1)/(2N_C) = 4/3$ and $C_A = N_C$ the Casimir operators of the fundamental and adjoint representations of the gauge group $SU(N_C)$, and $T_F = 1/2$ the normalisation of the generators of the fundamental representation.

²In $Z^0 \rightarrow q\bar{q}gg$ events the lowest energy parton is a gluon in $\approx 84\%$ of cases, whereas the percentage of events in which the two lowest energy partons are both gluons is only $\approx 53\%$ [14].

³An updated review on matrix element computations for multi-jet production in e^+e^- reactions presented in the literature can be found, e.g., in the introduction of ref. [23].

The matrix element squared for the two quark and two gluon process (2), can be split into two gauge invariant parts as follows [25]:

$$|\mathcal{M}|^2 = C_a^{q\bar{q}gg} |\mathcal{M}_a|^2 + C_b^{q\bar{q}gg} |\mathcal{M}_b|^2, \quad (7)$$

where

$$\mathcal{M}_a = \sum_{i=1,6} \mathcal{M}_i, \quad (8)$$

$$\mathcal{M}_b = \mathcal{M}_1 + \mathcal{M}_3 + \mathcal{M}_5 - \mathcal{M}_2 - \mathcal{M}_4 - \mathcal{M}_6 - 2i[\mathcal{M}_7 + \mathcal{M}_8], \quad (9)$$

\mathcal{M}_i , $i = 1, \dots, 8$, corresponds to the i -th diagram in fig. 2, and

$$C_a^{q\bar{q}gg} = \frac{1}{2} N_C C_F (2C_F - \frac{1}{2} C_A) = \frac{7}{3}, \quad C_b^{q\bar{q}gg} = \frac{1}{2} N_C C_F \frac{1}{2} C_A = 3. \quad (10)$$

The second term in eq. (7) is characteristic of non-Abelian theories and would be absent in any Abelian model.

Finally, for the production of two quarks, a gluon and a photon (3), the matrix element squared is (up to a constant factor, see later on):

$$|\mathcal{M}|^2 = C^{q\bar{q}g\gamma} |e_q \mathcal{M}_a + e_e \mathcal{M}_{ISR}|^2, \quad (11)$$

with

$$C^{q\bar{q}g\gamma} = N_C C_F, \quad (12)$$

where \mathcal{M}_a is the sum of the diagrams of fig. 2a, \mathcal{M}_{ISR} the sum of the diagrams of fig. 2c, and e_e and e_q are the electric charges of the electron e and of the quark q in the final state.

The Abelian model we compare with QCD is the one introduced in [26]⁴. Here gluons have no colour and no self-coupling: therefore, only diagrams of fig. 1 and fig. 2a survive. The cross sections of processes (1)-(2) for the Abelian case can be obtained from the QCD ones by simply replacing the group constants of QCD by those appropriate for the Abelian model: i.e., $C_A = 3 \rightarrow 0$, $C_F = 4/3 \rightarrow 1$ and $T_F = 1/2 \rightarrow 3$. The ‘‘Abelian coupling constant’’ α_A is fixed to $(4/3)\alpha_s$, so that the ratio of the two-jet and three-jet cross sections agrees with experiment.

We have analyzed the processes (1)-(3) adopting four different jet-finding algorithms. They are identified through their clustering variable y_{ij} . They are the JADE scheme (J) [28] based on the variable

$$y_{ij}^J = \frac{2E_i E_j (1 - \cos \theta_{ij})}{s}, \quad (13)$$

and its ‘‘E’’ variation (E)⁵

$$y_{ij}^E = \frac{(p_i + p_j) \cdot (p_i + p_j)}{s}, \quad (14)$$

the Durham scheme (D) [29]

$$y_{ij}^D = \frac{2 \min(E_i^2, E_j^2) (1 - \cos \theta_{ij})}{s}, \quad (15)$$

⁴Of course, we know that such a model has been already ruled out in other contexts, e.g., by measurements of the energy dependence of multi-jet production rates in e^+e^- annihilation [27], but we regard it mainly as a useful tool to demonstrate the sensitivity of the introduced angular variables to the various features of QCD.

⁵At lowest order, the E and JADE schemes are equivalent for massless particles.

and the Geneva algorithm (G) [30]

$$y_{ij}^G = \frac{8}{9} \frac{E_i E_j (1 - \cos \theta_{ij})}{(E_i + E_j)^2}. \quad (16)$$

For all of them the two (pseudo)particles i and j (with energy E_i and E_j , respectively) for which y_{ij} is minimum are combined into a single pseudoparticle k of momentum P_k given by the formula

$$P_k = P_i + P_j. \quad (17)$$

The procedure is iterated until all pseudoparticle pairs satisfy $y_{ij} \geq y_{cut}$. The various characteristics of these algorithms are well summarized in ref. [30]. In our lowest order calculation, the four jet cross section for a given algorithm is simply equal to the four parton cross section with a cut $y_{ij} \geq y_{cut}$ on all pairs of partons (i, j) .

Concerning the numerical part of our work, we have taken $\alpha_{em} = 1/128$ and $\sin^2 \theta_W \equiv s_W^2 = 0.23$, while for the Z^0 boson mass and width we have adopted the values $M_{Z^0} = 91.1$ GeV and $\Gamma_{Z^0} = 2.5$ GeV, respectively. For the quarks we have: $m_c = 1.7$ GeV and $m_b = 5.0$ GeV while the flavours u , d and s have been considered massless. Finally, the strong coupling constant α_s has been set equal to 0.115.

3 Angular variables

We study the following four variables: the modified Nachtmann-Reiter angle, θ_{NR}^* , the Bengtsson-Zerwas angle, χ_{BZ} , a modification of the Körner-Schierholz-Willrodt angle we denote by Φ_{KSW}^* , and the angle between jets 3 and 4, θ_{34} , in two different situations: (a) with b -tagging and (b) without b -tagging.

In case (a), we consider four-jet events where two of the jets contain a b or a \bar{b} . Let us call them jet 1 and jet 2. It does not matter which one is the b - and which the \bar{b} -jet. We do not make any assumptions about jets 3 and 4; they may be either gluon or quark jets, or even b -jets. Then, in terms of the three-momenta $\vec{p}_1, \dots, \vec{p}_4$ of jets 1, \dots , 4, the angles θ_{NR}^* , χ_{BZ} and θ_{34} are defined by

$$\theta_{NR}^* = \angle(\vec{p}_1 - \vec{p}_2, \vec{p}_3 - \vec{p}_4), \quad (18)$$

$$\chi_{BZ} = \angle(\vec{p}_1 \times \vec{p}_2, \vec{p}_3 \times \vec{p}_4), \quad (19)$$

and

$$\theta_{34} = \angle(\vec{p}_3, \vec{p}_4). \quad (20)$$

For events where

$$|\vec{p}_1 + \vec{p}_3| > |\vec{p}_1 + \vec{p}_4| \quad (21)$$

we define

$$\Phi_{KSW}^* = \angle(\vec{p}_1 \times \vec{p}_3, \vec{p}_2 \times \vec{p}_4). \quad (22)$$

In the opposite case, we define Φ_{KSW}^* with \vec{p}_3 and \vec{p}_4 interchanged. The definition in eqs. (21)-(22) is equivalent to the original definition of Φ_{KSW} [13] in events where the thrust axis is along $\vec{p}_1 + \vec{p}_3$ or $\vec{p}_1 + \vec{p}_4$.

In situation (b), where there is no b -tagging, we label the jets according to their energy, such that $E_1 \geq E_2 \geq E_3 \geq E_4$, and then define the angles by eqs. (18)-(22) as before.

By considering the polarization of the gluon in $e^+e^- \rightarrow q\bar{q}g$ and the final state helicities in the subsequent splitting $g \rightarrow gg$ or $g \rightarrow q'\bar{q}'$, one finds that the $e^+e^- \rightarrow q\bar{q}gg$ cross section is concentrated near $\cos \theta_{NR}^* \approx \pm 1$, whereas in $e^+e^- \rightarrow q\bar{q}q'\bar{q}'$, the cross section is largest around $\cos \theta_{NR}^* \approx 0$. In the case of the Bengtsson-Zerwas angle, one expects the $g \rightarrow gg$ contribution to be rather flat in the corresponding distribution if compared with the $g \rightarrow q\bar{q}$ one, which generally peaks at $\chi_{BZ} \approx 90^\circ$. The original Körner-Schierholz-Willrodt angle is defined for events for which there are two jets in both the hemispheres separated by the plane perpendicular to the thrust axis: it is the angle between the oriented normals to the plane containing the jets in one hemisphere and to the plane defined by the two other jets. The advantage of the modified definition Φ_{KSW}^* adopted here is that it allows us to include the complete 4jet-sample in the analysis, without having to discard events with three vectors in the same hemisphere. In the splitting process $g \rightarrow gg$, the two planes tend to be parallel, with the two gluons on the same side, i.e., $\Phi_{KSW}^{(*)} \approx \pi$, whereas the planes are preferentially orthogonal for $g \rightarrow q\bar{q}$. Finally, gluons from the triple gluon vertex $g \rightarrow gg$ and the second pair of quarks from $g \rightarrow q\bar{q}$ are expected to be closer together than gluons from double bremsstrahlung, and this should be evident by looking at the angle between the two softest jets. More details on these arguments are given in ref. [14].

The results for the Nachtmann-Reiter angle are shown in figs. 3a & b. In the case of b -tagging (fig. 3a), the distributions are even functions of $\cos \theta_{NR}^*$, because replacing θ_{NR}^* by $\pi - \theta_{NR}^*$ is equivalent to interchanging \vec{p}_3 and \vec{p}_4 , and we do not make any distinction between jets 3 and 4. The $b\bar{b}gg$ -distributions have peaks at $\cos \theta_{NR}^* = \pm 1$, whereas the maxima of the $b\bar{b}q\bar{q}$ are at $\cos \theta_{NR}^* = 0$. We also note that the $b\bar{b}gg$ -distributions in QCD and in the Abelian model are different. Without b -tagging, the distributions of $\cos \theta_{NR}^*$ (fig. 3b) are skewed to the left. The reason for this asymmetry is kinematical and can be understood by considering events where all four jets are all close to one common axis (with small angles between them in order to satisfy the y -cuts). Then, by momentum conservation, the most and the next most energetic jets, 1 and 2, must go in opposite directions. If we further restrict our attention to events where jets 3 and 4 also go in opposite directions, then energy-ordering and momentum conservation together imply that jet 3 must be parallel to jet 2, which gives $\theta_{NR} = \pi$, and cannot be parallel to jet 1, which would give $\theta_{NR} = 0$. Although the $q\bar{q}gg$ and $q\bar{q}q'\bar{q}'$ -distributions are still different, the difference between QCD and the Abelian model in the $q\bar{q}gg$ -distribution is washed away.

In figs. 4a & b, the distributions are shown of the Bengtsson-Zerwas angle, with and without b -tagging, respectively. With b -tagging the distributions are again symmetric, for the same reason as in the case of Nachtmann-Reiter. At $\chi_{BZ} = \pi/2$, there is a peak in the $b\bar{b}q\bar{q}$ -distributions and a dip in the $b\bar{b}gg$ -distributions. Without b -tagging, the $b\bar{b}gg$ -distributions are shifted to lower values of χ_{BZ} , whereas the $b\bar{b}q\bar{q}$ -distributions are shifted slightly towards higher χ_{BZ} .

The distributions where the rewards for b -tagging are largest are probably those of the angle Φ_{KSW}^* , shown in figs. 5a & b. With b -tagging, the differences between the $b\bar{b}gg$ distributions in QCD and the Abelian model become even more clear than in χ_{BZ} . In this respect the modified definition of Φ_{KSW}^* , initially adopted in order to avoid loss of statistics, turns out to

be extremely successful.

Finally, figs. 6a & b show the distribution of $\cos \theta_{34}$. Here we see, as expected, a tendency for the gluons from the triple gluon vertex, and the quarks $q\bar{q}$ in $b\bar{b}q\bar{q}$ events, to be closer together than the gluons in the Abelian model. In fig. 6a, there is also a large concentration of $b\bar{b}q\bar{q}$ events near $\cos \theta_{34} = -1$. They come from the region of phase space where the b -quarks are relatively soft, where the cross section is dominated by the diagrams in which the Z^0 is coupled directly to the quarks $q\bar{q}$. That explains why the peak disappears completely when the jets are energy-ordered (fig. 6b).

We should warn the reader that the value of y_{cut} we have chosen for using with the Geneva algorithm corresponds to a looser cut than the ones we use with the other algorithms. This is because, unlike the other jet defining variables, the definition (16) of y^G does not contain s explicitly, and therefore, the Geneva algorithm allows the energies of the partons to be much smaller for a given value of y_{cut} . As a result, we can get very close to the singularities of the matrix elements, where we expect radiative corrections to be large (figs. 5 and 6). The large peaks in the cross section also make it more difficult to integrate by VEGAS, as can be seen from the statistical fluctuations in the distributions of figs. 3 & 4. Therefore, a larger value of y_{cut}^G would be needed to obtain reliable predictions from our tree-level calculation [30], but we prefer to keep the value shown just to illustrate what happens.

4 Photon sample

In the literature [15, 16], it has been argued that one might be able to see the non-Abelian structure of QCD by comparing the distributions of four-jet events with those of events where three jets and a photon are produced. Since the diagrams of fig. 2a are the same, up to a constant factor, for the processes (2) and (3), any differences in their distributions must be due to the non-Abelian diagrams of fig. 2b and the ISR diagrams of fig. 2c. As we shall show below, the contribution of the latter can be made negligibly small by applying suitable cuts. If one then assumes that the four-jet events are predominantly $q\bar{q}gg$ events, as is true in QCD, differences between the four-jet and the three-jet plus photon distributions can be regarded as evidence for the non-Abelian contribution $|\mathcal{M}_b|^2$ in eq. (7).

What effect would b -tagging have on such an analysis? Presumably, the distinctions between the distributions would become more clear, but the number of events would be smaller. Moreover, selecting events with a $b\bar{b}$ -pair in the final state increases the relative number of unwanted four-quark events, since five of the fifteen flavour combinations $q\bar{q}q'\bar{q}'$, but only one of the five combinations $q\bar{q}gg$ (where $q, q' = u, d, c, s, b$), contain at least one $b\bar{b}$ -pair. It is possible to reduce this contamination of the four-jet sample by imposing cuts, at the cost of a further loss of statistics. The crucial question is, whether, in the end, the event rates would still be large enough to allow a study of the distributions.

The ISR diagrams are important for photons that are either soft or nearly collinear with the incoming electrons and positrons. Therefore, we can eliminate them by imposing cuts on the photon energy E_γ and on the angle $\theta_{beam-\gamma}$ between the photon trajectory and the e^\pm beams. Since we wish to compare the sample of three-jet plus photon events with an equivalent sample of four-jet events, we must treat both samples on exactly the same footing. In the four-jet sample we impose the same cuts on all four jets, or at least on the two non- b jets, in case we

select events where two of the jets are tagged as b -jets. This implies that in the three-jet plus photon sample, we must also impose the same cuts on the jet energies E_{jet} and their angles with respect to the beams $\theta_{beam-jet}$. In the b -tagged case, we only apply these cuts to the non- b jet.

It turns out that demanding that $|\cos \theta_{beam-\gamma}| < 0.9$ and $E_\gamma > 10$ GeV is sufficient to remove the effect of the ISR diagrams. This is illustrated, for example, in fig. 7, where we have implemented the cut $|\cos \theta_{beam-\gamma}| < 0.9$ and plotted the differential distribution in the photon energy of the three-jet plus photon cross section twice: taking the ISR diagrams into account and omitting them. Above our 10 GeV cut on E_γ , the two distributions are the same. We have checked that, with these cuts, the distributions of other variables also look the same with and without ISR.

The total cross section for process (3) is given in tab. I for several values of y_{cut} . The effect of the ISR is never greater than a few percent.

In tab. II we show the $q\bar{q}gg$ and $q\bar{q}q'\bar{q}'$ components of the total $e^+e^- \rightarrow 4\text{jet}$ cross section. In tab. IIa, where we assume two b -jets are tagged, we have imposed two additional cuts, namely $|\cos \theta_{NR}^*| > 0.5$ and $\chi_{BZ} < 50^\circ$ or $\chi_{BZ} > 130^\circ$. As can be seen in figs. 3 & 4, this reduces the relative size of the $q\bar{q}q'\bar{q}'$ component, making it less than about 8% of the total four jet cross section for all values of y_{cut} shown in the table. We have not imposed these cuts in tab. IIb, which shows the results if one does not select b -tagged jets, because there the $q\bar{q}q'\bar{q}'$ component is already quite small without them.

A comparison between the $e^+e^- \rightarrow q\bar{q}gg$ and $e^+e^- \rightarrow q\bar{q}g\gamma$ cross sections is made in fig. 8. They are displayed as a function of y_{cut} for each of the jet-finding algorithms. The dotted curves marked “real” show the actual $q\bar{q}g\gamma$ cross section. The dashed curves marked “renormalised” show the $q\bar{q}g\gamma$ cross section multiplied by

$$\frac{\alpha_s C_a^{q\bar{q}gg}}{2\alpha_{em} e_q^2 C^{q\bar{q}g\gamma}}. \quad (23)$$

Apart from the small ISR effect, this is exactly the contribution of the QED-like graphs, i.e., the term $|\mathcal{M}_a|^2$ in eq. (7), to the $e^+e^- \rightarrow q\bar{q}gg$ cross section. The factor of two in the denominator is the symmetry factor needed to account for the two identical gluons in the $q\bar{q}gg$ final state. The non-Abelian contribution $|\mathcal{M}_b|^2$ is the difference between this “renormalised” $q\bar{q}g\gamma$ cross section and the $q\bar{q}gg$ cross section.

5 Mass effects

In this section, we examine the numerical importance of taking the b -quark mass into account exactly, as we have done in all the calculations we have discussed until now. The b -quark mass does not only affect the total cross section [22, 23, 24], but also some of the angular distributions. In figs. 9, 10 & 11, we present plots illustrating this for the processes $e^+e^- \rightarrow b\bar{b}gg$, $e^+e^- \rightarrow b\bar{b}u\bar{u}$, and $e^+e^- \rightarrow b\bar{b}b\bar{b}$. We do not show, e.g., $e^+e^- \rightarrow b\bar{b}d\bar{d}$, because the results are quite similar to those of $e^+e^- \rightarrow b\bar{b}u\bar{u}$. In each case, we compare the distributions obtained for $m_b = 5$ GeV with the ones we would find if we neglected m_b .

The differences turn out to be fairly small in the $b\bar{b}gg$ -process, but in the other ones they are quite large, particularly in the distributions of χ_{BZ} and Φ_{KSW}^* . This is true both for the

distributions where the b -jets are identified, see figs. 10a & 11a, and for the distributions where the jets are simply ordered in energy, figs. 10b and 11b. These mass effects also depend on the particular jet-finding algorithm used. As an example we plotted the case of the E scheme, for which the mass effects are larger.

In the equal flavour process, $e^+e^- \rightarrow b\bar{b}b\bar{b}$, there are three different combinations of b -(anti)quarks that can be tagged: two b 's, one b and one \bar{b} , or two \bar{b} 's. That is why fig. 11a shows two sets of curves, one for the $b\bar{b}$ case and one for the $b\bar{b}$ case⁶. We also note that even in the $b\bar{b}$ case, the distribution of $\cos\theta_{NR}^*$ is very different from the corresponding distribution in the unequal flavour process. Due to the gluon propagators in the last four diagrams of fig. 1, it peaks at $\cos\theta_{NR}^* = \pm 1$, making it look similar to the $\cos\theta_{NR}^*$ distribution of the $b\bar{b}gg$ process.

6 Conclusions

We have given our results in terms of total and differential cross sections. To convert them into event rates, they must be multiplied by the luminosity and the efficiency for tagging two b -quarks. We have not done this because the numbers differ from one experiment to another, and could still change as the techniques are improved. However, to get an idea of what one might expect, let us suppose the integrated luminosity is 100 pb^{-1} and the tagging efficiency 50%. Then there will be roughly 4×10^4 four jet events with two tagged b -quarks (using the JADE scheme with $y_{cut} = 0.01$, as in figs. 3-6). This is an order of magnitude less than the total number of four jet events, but in return we gain greater power to discriminate between the various terms in the four jet cross section, particularly between the Abelian, QED-like term and the non-Abelian term in the two quark, two gluon cross section.

In a realistic analysis, one should take the probability of misidentifying other particles as b -quarks into account. We do not expect c -jets mistagged as b -jets to distort the distributions severely, although they would increase the total number of events, because they would still be correctly classified as quark jets. It is, of course, important that the number of gluon jets tagged as b 's be as small as possible. The probability of a b -quark being created during the fragmentation of a gluon or a lighter quark is believed to be negligible [19].

The number of three jet γ events is reduced even more severely by demanding two tagged b -quarks, because of their small electric charge ($-1/3$ instead of $2/3$). With the luminosity and efficiency given above, and the cuts of fig. 8a, one would expect of the order of 50 events, which is not enough to study any distributions. So, in the photon sample, it is better to keep all events.

Finally, we indicated the effects of the b -quark mass. Normally, they can safely be neglected because only a small fraction of the four jet events contain b -quarks, and moreover, most of those are $b\bar{b}gg$ events, where the effects are small. This is no longer true with b -tagging, because now all events contain at least two massive b -quarks, and the relative number of four quark events is higher, and the mass effects are especially important when comparing QCD with the Abelian model, where as many as 30% of the events are four quark events.

⁶In figs. 3a, 4a, 5a & 6a, we assumed a b and a \bar{b} were tagged. If that assumption is not true, the contribution of the $e^+e^- \rightarrow b\bar{b}b\bar{b}$ events to those distributions has to be replaced with a weighted average of the two sets of curves in fig. 11a. However, since the contribution of the $e^+e^- \rightarrow b\bar{b}b\bar{b}$ events is small, the distributions in figs. 3a, 4a, 5a & 6a would hardly change.

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References

- [1] T. Hebbeker, *Phys. Rep.* **217** (1992) 69.
- [2] L3 Collaboration, B. Adeva *et al.*, *Phys. Lett.* **B248** (1990) 464;
OPAL Collaboration, M.Z. Akrawy *et al.*, *Phys. Lett.* **B235** (1990) 389;
OPAL Collaboration, M.Z. Akrawy *et al.*, *Z. Phys.* **C 49** (1991) 375;
ALEPH Collaboration, D. Decamp *et al.*, *Phys. Lett.* **B255** (1991) 623;
DELPHI Collaboration, P. Abreu *et al.*, *Phys. Lett.* **B247** (1990) 167;
DELPHI Collaboration, P. Abreu *et al.*, *Z. Phys.* **C 54** (1992) 55;
ALEPH Collaboration, D. Decamp *et al.*, *Phys. Lett.* **B284** (1992) 163;
OPAL Collaboration, P.D. Acton *et al.*, *Z. Phys.* **C55** (1992) 1;
ALEPH Collaboration, D. Buskulic *et al.*, *Z. Phys.* **C55** (1992) 39;
ALEPH Collaboration, D. Buskulic *et al.*, *Z. Phys.* **C55** (1992) 209;
OPAL Collaboration, P.D. Acton *et al.*, *Z. Phys.* **C59** (1993) 1.
- [3] L3 Collaboration, B. Adeva *et al.*, *Phys. Lett.* **B271** (1991) 461;
DELPHI Collaboration, P. Abreu *et al.*, *preprint* CERN-PPE/93-59.
- [4] MARK II Collaboration, S. Bethke *et al.*, *Z. Phys.* **C43** (1989) 325;
TASSO Collaboration, W. Braunschweig *et al.*, *Phys. Lett.* **B214** (1988) 286;
AMY Collaboration, I.H. Park *et al.*, *Phys. Rev. Lett.* **62** (1989) 1713;
VENUS Collaboration, K. Abet *et al.*, *Phys. Lett.* **B240** (1990) 232;
ALEPH Collaboration, D. Decamp *et al.*, *Phys. Lett.* **B255** (1991) 623;
DELPHI Collaboration, P. Abreu *et al.*, *Phys. Lett.* **B247** (1990) 167;
L3 Collaboration, B. Adeva *et al.*, *Phys. Lett.* **B248** (1990) 464;
OPAL Collaboration, M.Z. Akrawy *et al.*, *Phys. Lett.* **B235** (1990) 389;
OPAL Collaboration, M.Z. Akrawy *et al.*, *Z. Phys.* **C49** (1991) 375.
- [5] L3 Collaboration, B. Adeva *et al.*, *Phys. Lett.* **B263** (1991) 551;
DELPHI Collaboration, P. Abreu *et al.*, *Phys. Lett.* **B274** (1992) 498;
OPAL Collaboration, G. Alexander *et al.*, *Z. Phys.* **C52** (1991) 543.
- [6] L3 Collaboration, B. Adeva *et al.*, *Phys. Lett.* **B248** (1990) 227;
OPAL Collaboration, M.Z. Akrawy *et al.*, *Z. Phys.* **C49** (1991) 49.
- [7] DELPHI Collaboration, P. Abreu *et al.*, *Phys. Lett.* **B255** (1991) 466;
ALEPH Collaboration, D. Decamp *et al.*, *Phys. Lett.* **B284** (1992) 151;
DELPHI Collaboration, P. Abreu *et al.*, *Z. Phys.* **C59** (1993) 357.

- [8] L3 Collaboration, B. Adeva *et al.*, *Phys. Lett.* **B248** (1990) 227;
OPAL Collaboration, M.Z. Akrawy *et al.*, *Z. Phys.* **C49** (1991) 49.
- [9] J. Ellis and I. Karliner, *Nucl. Phys.* **B148** (1979) 141.
- [10] O. Nachtmann and A. Reiter, *Z. Phys.* **C16** (1982) 45.
- [11] M. Bengtsson, *Z. Phys.* **C42** (1989) 75.
- [12] M. Bengtsson and P.M. Zerwas, *Phys. Lett.* **B208** (1988) 306.
- [13] J.G. Körner, G. Schierholz and J. Willrodt, *Nucl. Phys.* **B185** (1981) 365.
- [14] S. Bethke, A. Ricker and P.M. Zerwas, *Z. Phys.* **C49** (1991) 59.
- [15] Proceedings of the Workshop on Photon Radiation from Quarks, S. Cartwright ed., An-necy, France, 2-3 December. 1991. CERN 92-04.
- [16] P. Mättig, in ref. [15].
- [17] See for example:
ALEPH Collaboration, D. Buskulic *et al.*, *Phys. Lett.* **B313** (1993) 535;
ALEPH Collaboration, D. Buskulic *et al.*, *Phys. Lett.* **B313** (1993) 549;
ALEPH Collaboration, D. Buskulic *et al.*, *Z. Phys.* **C62** (1994) 179;
ALEPH Collaboration, D. Buskulic *et al.*, *preprint* CERN-PPE/94-023, January 1994;
DELPHI Collaboration, P. Abreu *et al.*, *Phys. Lett.* **B295** (1992) 383;
DELPHI Collaboration, E.H. Cortina *et al.*, *preprint* LPNHE-92-05, DELPHI-92-27-PHYS-163, March 1992;
L3 Collaboration, O. Adriani *et al.*, *Phys. Lett.* **B292** (1992) 454;
L3 Collaboration, O. Adriani *et al.*, *Phys. Lett.* **B307** (1993) 237;
OPAL Collaboration, P.D. Acton *et al.*, *Z. Phys.* **C58** (1993) 387;
OPAL Collaboration, R. Akers *et al.*, *preprint* CERN-PPE-94-106, July 1994.
- [18] S. Squarcia, *preprint* CERN-PPE-94-69, April 1994, Invited talk at the “22nd INS Inter-national Symposium on Physics with High Energy Colliders”, Tokyo, Japan, 8-10 March 1994, and references therein.
- [19] A. De Angelis, *preprint* UDPHIR 95/01/AA, January 1995.
- [20] Z. Fodor, *Phys. Rev.* **D41** (1990) 1726, and references therein.
- [21] A. Vorvolakos, *preprint* PRINT-93-0768, November 1993, Seminar presented at the Impe-rial Coll. HEP Group, 17 November 1993, and references therein.
- [22] A. Ballestrero, E. Maina and S. Moretti, *Phys. Lett.* **B294** (1992) 425.
- [23] A. Ballestrero, E. Maina and S. Moretti, *Nucl. Phys.* **B415** (1994) 265.
- [24] A. Ballestrero, E. Maina and S. Moretti, *University of Torino preprint* DFTT 14/94, April 1994 (To be published in *Proceedings of the XXIXth Rencontres de Moriond, Méribel, Savoie, France, March 1994*).

- [25] K. Hagiwara and D. Zeppenfeld, *Nucl. Phys.* **B313** (1989) 560.
- [26] K.J.F. Gaemers and J.A.M. Vermaseren, *Z. Phys.* **C7** (1980) 81;
O. Nachtmann and A. Reiter, *Z. Phys.* **C16** (1982) 45.
- [27] OPAL Collaboration, M.Z. Akrawy, *Phys. Lett.* **B235** (1990) 389;
S. Bethke, in *Proceedings of the XXVth Rencontres de Moriond, Les Arcs, Savoie, France, March 1990*;
S. Bethke, *preprint* Heidelberg HD-PY 90/3;
J. Drees, in *Proceedings of the Neutrino-90 Conference, CERN, Geneva, Switzerland, June 1990*.
- [28] JADE Collaboration, W. Bartel *et al.*, *Z. Phys.* **C33** (1986) 23;
JADE Collaboration, S. Bethke *et al.*, *Phys. Lett.* **B213** (1988) 235.
- [29] N. Brown and W.J. Stirling, *Phys. Lett.* **B252** (1990) 657;
N. Brown and W.J. Stirling, *Z. Phys.* **C 53** (1992) 629.
- [30] S. Bethke, Z. Kunszt, D.E. Soper and W.J. Stirling, *Nucl. Phys.* **B370** (1992) 310.

Table Captions

tab. I Cross sections in picobarns of the processes (a) $e^+e^- \rightarrow b\bar{b}g\gamma$ (i.e., b -tagging) and (b) $e^+e^- \rightarrow \sum_q q\bar{q}g\gamma$ (i.e., no b -tagging), with and without ISR, for three different values of y_{cut} of each jet-finding algorithm, using the notation $(\sigma_{ISR}, \sigma_{noISR}; y_{cut})$.

The following additional cuts have been implemented: $|\cos \theta_{\text{beam}-\gamma, g}| < 0.9$ and $E_{\gamma, g} > 10$ GeV, for case (a), and $|\cos \theta_{\text{beam}-\gamma, g, q, \bar{q}}| < 0.9$ and $E_{\gamma, g, q, \bar{q}} > 10$ GeV, for case (b).

tab. II Cross sections in picobarns of the processes (a) $e^+e^- \rightarrow b\bar{b}gg$ (x) and $e^+e^- \rightarrow \sum_q b\bar{b}q\bar{q}$ (y) (i.e., b -tagging) and (b) $e^+e^- \rightarrow \sum_q q\bar{q}gg$ (x) and $e^+e^- \rightarrow \sum_q \sum_{q'} q\bar{q}q'\bar{q}'$ (y) (i.e., no b -tagging), for three different values of y_{cut} (z) of each jet-finding algorithm, adopting the notation $(x, y; z)$.

The following additional cuts have been implemented: $|\cos \theta_{\text{beam}-g}| < 0.9$, $E_g > 10$ GeV, $|\cos \theta_{NR}^*| > 0.5$ and $\chi_{BZ} < 50^\circ$ or $\chi_{BZ} > 130^\circ$, for case (a), and $|\cos \theta_{\text{beam}-g, q, \bar{q}}| < 0.9$ and $E_{g, q, \bar{q}} > 10$ GeV, for case (b).

Figure Captions

- fig. 1** Feynman diagrams contributing in lowest order to $e^+e^- \rightarrow q\bar{q}q'\bar{q}'$, where $q^{(\prime)} = u, d, s, c$ and b . If $q \neq q'$ only the first four diagrams contribute. The internal wavy line represents a photon or a Z^0 . The particles are labelled as in eq. (1).
- fig. 2** Feynman diagrams contributing in lowest order to $e^+e^- \rightarrow q\bar{q}gg$ (a and b) and $e^+e^- \rightarrow q\bar{q}g\gamma$ (a and c), where $q = u, d, s, c$ and b . The internal wavy line represents a photon or a Z^0 , while the external jagged line represents a gluon or a photon, as appropriate. The particles are labelled as in eqs. (2)-(3).
- fig. 3** Distributions in the cosine of the modified Nachtmann-Reiter angle, $\cos\theta_{NR}^*$, in (a) $e^+e^- \rightarrow b\bar{b}gg, \sum_q b\bar{b}q\bar{q}$ (i.e., b -tagging) and (b) $e^+e^- \rightarrow \sum_q q\bar{q}gg, \sum_q \sum_{q'} q\bar{q}q'\bar{q}'$ (i.e., no b -tagging), for the various jet-finding algorithms.
- fig. 4** Distributions in the Bengtsson-Zerwas angle, χ_{BZ} , in (a) $e^+e^- \rightarrow b\bar{b}gg, \sum_q b\bar{b}q\bar{q}$ (i.e., b -tagging) and (b) $e^+e^- \rightarrow \sum_q q\bar{q}gg, \sum_q \sum_{q'} q\bar{q}q'\bar{q}'$ (i.e., no b -tagging), for the various jet-finding algorithms.
- fig. 5** Distributions in the modified Körner-Schierholz-Willrodt angle, Φ_{KSW}^* , in (a) $e^+e^- \rightarrow b\bar{b}gg, \sum_q b\bar{b}q\bar{q}$ (i.e., b -tagging) and (b) $e^+e^- \rightarrow \sum_q q\bar{q}gg, \sum_q \sum_{q'} q\bar{q}q'\bar{q}'$ (i.e., no b -tagging), for the various jet-finding algorithms.
- fig. 6** Distributions in the cosine of the angle between the vectors \vec{p}_3 and \vec{p}_4 , $\cos\theta_{34}$, in (a) $e^+e^- \rightarrow b\bar{b}gg, \sum_q b\bar{b}q\bar{q}$ (i.e., b -tagging) and (b) $e^+e^- \rightarrow \sum_q q\bar{q}gg, \sum_q \sum_{q'} q\bar{q}q'\bar{q}'$ (i.e., no b -tagging), for the various jet-finding algorithms.
- fig. 7** Distributions in energy of the photon E_γ in (a) $e^+e^- \rightarrow b\bar{b}g\gamma$ (i.e., b -tagging), with $|\cos\theta_{\text{beam}-\gamma,g}| < 0.9$ and $E_{\gamma,g} > 1.0$ GeV, and (b) $e^+e^- \rightarrow \sum_q q\bar{q}g\gamma$ (i.e., no b -tagging), with $|\cos\theta_{\text{beam}-\gamma,g,q,\bar{q}}| < 0.9$ and $E_{\gamma,g,q,\bar{q}} > 1.0$ GeV, with and without ISR, for the various jet-finding algorithms.
- fig. 8** Cross sections of the processes (a) $e^+e^- \rightarrow b\bar{b}gg, b\bar{b}g\gamma$ (the latter both real and renormalized to the former, see in the text) (i.e., b -tagging), with $|\cos\theta_{\text{beam}-\gamma,g}| < 0.9$ and $E_{\gamma,g} > 10$ GeV, and (b) $e^+e^- \rightarrow \sum_q q\bar{q}gg, \sum_q q\bar{q}g\gamma$ (the latter both real and renormalized to the former, see in the text) (i.e., no b -tagging), with $|\cos\theta_{\text{beam}-\gamma,g,q,\bar{q}}| < 0.9$ and $E_{\gamma,g,q,\bar{q}} > 10$ GeV, as a function of y_{cut} for the various jet-finding algorithms. The following angular cuts have been also implemented in case (a): $|\cos\theta_{NR}^*| > 0.5$ and $\chi_{BZ} < 50^\circ$ or $\chi_{BZ} > 130^\circ$.
- fig. 9** Mass effects in the angular distributions of the process $e^+e^- \rightarrow b\bar{b}gg$ using the E jet finding algorithm. In (a) b -tagging is assumed, in (b) the jets are energy-ordered. The curves denoted by $b\bar{b}gg$ are for massive, and those denoted by $d\bar{d}gg$ for massless b -quarks.
- fig. 10** Mass effects in the angular distributions of the process $e^+e^- \rightarrow b\bar{b}u\bar{u}$ ($q \neq b$), using the E jet-finding algorithm. In (a) b -tagging is assumed, in (b) the jets are energy-ordered. The curves denoted by $b\bar{b}q\bar{q}$ are for massive, and those denoted by $d\bar{d}q\bar{q}$ for massless b -quarks.

fig. 11 Mass effects in the angular distributions of the process $e^+e^- \rightarrow b\bar{b}b\bar{b}$, using the E jet-finding algorithm. (a), solid and short-dashed lines: one b and one \bar{b} are tagged; dotted and long-dashed lines: two b 's are tagged. In (b), the jets are energy-ordered. The curves denoted by $b\bar{b}b\bar{b}$ are for massive, and those denoted by $d\bar{d}d\bar{d}$ for massless b -quarks.

J	E	D	G
(1.76, 1.68; 0.01)	(2.39, 2.28; 0.01)	(2.72, 2.62; 0.0015)	(2.64, 2.54; 0.0015)
(1.08, 1.02; 0.02)	(1.38, 1.32; 0.02)	(2.26, 2.17; 0.0030)	(2.12, 2.03; 0.0030)
(0.68, 0.65; 0.03)	(0.84, 0.80; 0.03)	(1.92, 1.83; 0.0045)	(1.75, 1.67; 0.0045)

Table Ia

J	E	D	G
(15.08, 14.73; 0.01)	(15.78, 15.41; 0.01)	(31.27, 30.70; 0.0015)	(27.66, 27.13; 0.0015)
(8.49, 8.27; 0.02)	(8.77, 8.54; 0.02)	(22.38, 21.91; 0.0030)	(19.31, 18.89; 0.0030)
(5.21, 5.06; 0.03)	(5.36, 5.20; 0.03)	(17.66, 17.28; 0.0045)	(14.93, 14.58; 0.0045)

Table Ib

J	E	D	G
(89.60, 5.93; 0.01)	(128.44, 10.78; 0.01)	(165.26, 10.66; 0.0015)	(155.97, 10.00; 0.0015)
(47.51, 3.19; 0.02)	(63.23, 5.73; 0.02)	(126.71, 8.11; 0.0030)	(114.68, 7.50; 0.0030)
(26.70, 1.79; 0.03)	(33.98, 2.82; 0.03)	(101.64, 6.45; 0.0045)	(89.16, 5.89; 0.0045)

Table IIa

J	E	D	G
(761.23, 39.08; 0.01)	(809.96, 44.94; 0.01)	(1618.04, 75.85; 0.0015)	(1410.82, 64.72; 0.0015)
(419.70, 23.91; 0.02)	(440.26, 26.95; 0.02)	(1152.07, 58.14; 0.0030)	(980.17, 48.20; 0.0030)
(254.69, 15.60; 0.03)	(265.77, 17.25; 0.03)	(910.63, 48.14; 0.0045)	(752.54, 38.85; 0.0045)

Table IIb

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